



เฉลยพิชิตข้อสอบ PAT 1 เรื่อง จำนวนเชิงซ้อน

ข้อ 8 หน้า 4 ตอบ 2

$$\text{ให้ } z = a + bi$$

$$|z| + 2\bar{z} - 3z = 3 - 45i \rightarrow |a + bi| + 2(a - bi) - 3(a + bi) = 3 - 45i$$

$$(\sqrt{a^2 + b^2} - a) - 5bi = 3 - 45i \text{ จากการเท่ากันของจำนวนเชิงซ้อนจะได้ว่า}$$

$$-5b = -45 \rightarrow b = 9 \text{ และ } \sqrt{a^2 + 9^2} - a = 3 \rightarrow a^2 + 9^2 = (a + 3)^2$$

$$a^2 + 81 = a^2 + 6a + 9 \rightarrow a = 12 \text{ ดังนั้น } z = a + bi = 12 + 9i$$

$$\therefore |\bar{z}|^2 = |z|^2 = 12^2 + 9^2 = 225$$

ข้อ 9 หน้า 5 ตอบ 4

$$\text{ให้ } z = a + bi$$

$$\bar{z} - 1 - 4i = 3i(z - i) \rightarrow a - bi - 1 - 4i = 3i(a + bi - i)$$

$$a - 1 - bi - 4i = 3ai - 3b + 3 \rightarrow (a + 3b) - (b + 3a)i = 4 + 4i$$

$$a + 3b = 4 \text{ —————(1) , } b + 3a = -4 \text{ —————(2)}$$

$$\text{จาก (1) และ (2) จะได้ } a = -2, b = 2$$

$$\text{ดังนั้น } z = -2 + 2i, \bar{z} = -2 - 2i$$

$$\text{ข้อ 1) } z + \bar{z} = -4, i(z - \bar{z}) = i(4i) = 4i^2 = -4 \therefore \text{ข้อ 1 ถูกต้อง}$$

$$\text{ข้อ 2) } |z + 2| = |-2 + 2i + 2| = |2i| = 2 \therefore \text{ข้อ 2 ถูกต้อง}$$

$$\text{ข้อ 3) } z^2 = [(-2 + 2i)]^2 = 8i \rightarrow z^2 - 8i = 0 \therefore \text{ข้อ 3 ถูกต้อง}$$

$$\begin{aligned} \text{ข้อ 4) } z(1 - i)^3 &= (-2 + 2i)(1 - i)^3 = -2(1 - i)(1 - i)^3 = -2(1 - i)^4 \\ &= -2(-2i)^2 = -2(-4) = 8 \therefore \text{ข้อ 4 ไม่ถูกต้อง} \end{aligned}$$

ข้อ 10 หน้า 5 ตอบ 5

$$\text{ให้ } z = a + bi$$

$$3z\bar{z} - (28 - i)z + 4z^2 = 0 \rightarrow z(3\bar{z} + 4z - (28 - i)) = 0$$

$$z(3\bar{z} + 4z - 28 + i) = 0$$

$$\text{กรณีที่ 1 } z = 0$$

$$\text{กรณีที่ 2 } 3\bar{z} + 4z - 28 + i = 0 \rightarrow 3(a - bi) + 4(a + bi) - 28 + i = 0$$

$$(7a - 28) + (b + 1)i = 0 \rightarrow a = 4, b = -1 \rightarrow a + bi = 4 - i$$

ดังนั้น $A = \{0, 4 - i\}$

จากโจทย์ $B = \{|z + i| / z \in A\}$
 $= \{|0 + i|, |4 - i + i|\} = \{1, 4\}$

$\therefore$ ผลบวกของสมาชิกทั้งหมดในเซต $B = 1 + 4 = 5$

ข้อ 11 หน้า 6

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + \bar{z}_1 z_2 + z_1 \bar{z}_2 \quad \text{---(1)}$$

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - \bar{z}_1 z_2 - z_1 \bar{z}_2 \quad \text{---(2)}$$

(1) + (2) จะได้ว่า

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

ข้อ 12 หน้า 6 ตอบ 3

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

$$\frac{5+1}{2} = |z_1|^2 + |z_2|^2 = 3$$

ข้อ 13 หน้า 7 ตอบ 5

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

$$\frac{3^2+1^2}{2} = |z_1|^2 + |z_2|^2 = 5$$

ข้อ 14 หน้า 7 ตอบ 3

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

$$|z_1 + z_2|^2 + 1^2 = 2((\sqrt{2})^2 + (\sqrt{3})^2) = 10$$

$$\therefore |z_1 + z_2| = 3$$

ข้อ 15 หน้า 8 ตอบ 1

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + \bar{z}_1 z_2 + z_1 \bar{z}_2$$

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + \overline{z_1 z_2} + z_1 \bar{z}_2$$

$$3^2 = |z_1|^2 + |z_2|^2 + (3 - 4i) + (3 + 4i)$$

$$\therefore |z_1|^2 + |z_2|^2 = 3$$

ข้อ 16 หน้า 8 ตอบ 2

$$z_1 + z_2 + z_3 = 0 \rightarrow z_1 + z_2 = -z_3 \rightarrow |z_1 + z_2| = |-z_3| = |z_3|$$

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

$$|z_3|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

$$1 + |z_1 - z_2|^2 = 2(1 + 1) = 4$$

$$|z_1 - z_2| = \sqrt{3}$$

ข้อ 17 หน้า 9 ตอบ 1

$$z_1 = a + bi, z_2 = c + di$$

$$z_1 - z_2 = s \rightarrow a + bi - (c + di) = s \rightarrow (a - c) + (b - d)i = s + 0i$$

$$\text{จากการเท่ากัน } b - d = 0 \rightarrow \boxed{b = d}$$

$$z_1^2 + z_2^2 = t \rightarrow a^2 + 2abi - b^2 + c^2 + 2cdi - d^2 = t \rightarrow (a^2 - b^2) + (c^2 - d^2) + 2(ab + cd)i = t + 0i$$

$$\text{จากการเท่ากัน } 2(ab + cd) = 0 \rightarrow ab + cd = 0 \rightarrow ab = -cd \rightarrow \boxed{a = -c} \quad (\because b = d)$$

$$\text{ดังนั้น } z_1 = a + bi = -c + di \rightarrow |z_1| = |-c + di| = \sqrt{c^2 + d^2} = |z_2| \quad \therefore (\text{ก}) \text{ ถูก}$$

$$z_1 z_2 = (-c + di)(c + di) = -(c - di)(c + di) = -(c^2 + d^2) + 0i$$

$$\text{จะได้ } \text{Im}(z_1 z_2) = 0 \quad \therefore (\text{ข}) \text{ ถูก}$$

ข้อ 18 หน้า 10 ตอบ 4

$$z^3 = i \rightarrow z = -i, \frac{\sqrt{3} + i}{2}, \frac{-\sqrt{3} + i}{2}$$

$$\text{แต่โจทย์กำหนด } z = a + bi \text{ โดยที่ } ab > 0 \text{ ดังนั้น } z = \frac{\sqrt{3} + i}{2}$$

$$\begin{aligned} \therefore |iz^5 + 2|^2 &= \left| i \frac{z^6}{z} + 2 \right|^2 = \left| i \frac{(z^3)^2}{z} + 2 \right|^2 = \left| i \frac{(i)^2}{z} + 2 \right|^2 \\ &= \left| \frac{i^3 + 2z}{z} \right|^2 = \frac{|-i + 2z|^2}{|z|^2} = \frac{|-i + \sqrt{3} + i|^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 3 \end{aligned}$$

ข้อ 19 หน้า 10 ตอบ 2

$$\text{จากโจทย์ } z^4 + 1 = 0$$

$$\therefore \left| z + \frac{1}{z} \right|^2 = \left| \left(z + \frac{1}{z} \right)^2 \right| = \left| z^2 + 2 + \frac{1}{z^2} \right| = \left| \frac{z^4 + 1}{z^2} + 2 \right| = |0 + 2| = 2$$

ข้อ 20 หน้า 11 ตอบ 5

$$\bar{z}_2 = 1+2i \rightarrow z_2 = 1-2i$$

$$5z_1 + 2z_2 = 5 \rightarrow 5z_1 + 2(1-2i) = 5 \rightarrow 5z_1 = 3+4i$$

$$5|z_1| = |3+4i| \rightarrow 5|z_1| = 5 \rightarrow |z_1| = 1 \quad \therefore |5z_1^{-1}| = |5||z_1^{-1}| = \frac{5}{|z_1|} = 5$$

ข้อ 21 หน้า 11 ตอบ 2

$$|z+a^2| = a|z+1| \quad \text{จะได้} \quad |z| = a$$

$$|z+2^2| = 2|z+1| \quad \text{จะได้} \quad |z| = 2 = |\bar{z}|$$

ข้อ 22 หน้า 12 ตอบ 3

$$|z-a^2| = a|z-1| \quad \text{จะได้} \quad |z| = a$$

$$|z-3^2| = 3|z-1| \quad \text{จะได้} \quad |z| = 3$$

$$\therefore z\bar{z} = |z|^2 = 3^2 = 9 \in [9, 13)$$

ข้อ 23 หน้า 12 ตอบ 4

$$\begin{aligned} |\overline{1+\cos\theta+i\sin\theta}| &= |1+\cos\theta+i\sin\theta| = \sqrt{(1+\cos\theta)^2 + \sin^2\theta} \\ &= \sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta} = \sqrt{2+2\cos\theta} \\ &= \sqrt{2+2\cos 2\left(\frac{\theta}{2}\right)} = \sqrt{2+2(2\cos^2\frac{\theta}{2}-1)} \\ &= \sqrt{4\cos^2\frac{\theta}{2}} = 2\left|\cos\frac{\theta}{2}\right| \end{aligned}$$

ข้อ 24 หน้า 13 ตอบ 1

$$\frac{1}{z} = \frac{a+ci+a+bi}{(a+bi)(a+ci)} = \frac{2a+(b+c)i}{(a+bi)(a+ci)}$$

$$z = \frac{(a+bi)(a+ci)}{2a+(b+c)i}$$

$$|z| = \frac{|a+bi||a+ci|}{|2a+(b+c)i|} \rightarrow |z|^2 = \frac{|a+bi|^2|a+ci|^2}{|2a+(b+c)i|^2}$$

$$z \cdot \bar{z} = \frac{(a^2+b^2)(a^2+c^2)}{4a^2+(b+c)^2}$$

ข้อ 27 หน้า 14 ตอบ 0

$$|z_1|^2 = 1 \rightarrow z_1 \bar{z}_1 = 1 \rightarrow \bar{z}_1 = \frac{1}{z_1}$$

$$|z_2|^2 = 1 \rightarrow z_2 \bar{z}_2 = 1 \rightarrow \bar{z}_2 = \frac{1}{z_2}$$

$$|z_3|^2 = 1 \rightarrow z_3 \bar{z}_3 = 1 \rightarrow \bar{z}_3 = \frac{1}{z_3}$$

$$(z_1 + z_2 + z_3)^2 = z_1^2 + z_2^2 + z_3^2 + 2(z_1 z_2 + z_2 z_3 + z_3 z_1)$$

$$0^2 = z_1^2 + z_2^2 + z_3^2 + 2\left(\frac{z_1 z_2 z_3}{z_3} + \frac{z_1 z_2 z_3}{z_1} + \frac{z_1 z_2 z_3}{z_2}\right)$$

$$0 = z_1^2 + z_2^2 + z_3^2 + 2z_1 z_2 z_3 \left(\frac{1}{z_3} + \frac{1}{z_1} + \frac{1}{z_2}\right)$$

$$0 = z_1^2 + z_2^2 + z_3^2 + 2z_1 z_2 z_3 (\bar{z}_3 + \bar{z}_1 + \bar{z}_2)$$

$$0 = z_1^2 + z_2^2 + z_3^2 + 2z_1 z_2 z_3 (\overline{z_1 + z_2 + z_3})$$

$$0 = z_1^2 + z_2^2 + z_3^2 + 2z_1 z_2 z_3 (\bar{0})$$

$$\therefore z_1^2 + z_2^2 + z_3^2 = 0$$

ข้อ 29 หน้า 15 ตอบ 2

$$\frac{|z_1 - 2z_2|}{|2 - z_1 \bar{z}_2|} = 1 \rightarrow |z_1 - 2z_2| = |2 - z_1 \bar{z}_2| \rightarrow |z_1 - 2z_2|^2 = |2 - z_1 \bar{z}_2|^2$$

$$|z_1|^2 + |2z_2|^2 - \bar{z}_1(2z_2) - z_1(\overline{2z_2}) = |2|^2 + |z_1 \bar{z}_2|^2 - \bar{z}_1(z_1 \bar{z}_2) - 2(\overline{z_1 \bar{z}_2})$$

$$|z_1|^2 + 4|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1 \bar{z}_2 = 4 + |z_1|^2 |\bar{z}_2|^2 - 2z_1 \bar{z}_2 - 2\bar{z}_1 z_2$$

$$|z_1|^2 - |z_1|^2 |\bar{z}_2|^2 = 4 - 4|z_2|^2$$

$$|z_1|^2 - |z_1|^2 |z_2|^2 = 4 - 4|z_2|^2$$

$$|z_1|^2 (1 - |z_2|^2) = 4(1 - |z_2|^2)$$

$$|z_1|^2 = 4 \quad \therefore |z_1| = 2$$

ขอให้ทุกคนโชคดีในการสอบนะครับ
ด้วยความห่วงใยจากใจจริง

พี่ช้าง

